

6) CONVERTING PARAMETRIC EQUATIONS OF PARABOLA TO CARTESIAN FORM – by eliminating the t variable.

1) Find the Cartesian equation for parabolas defined by these parametric equations:

a) $x = 3t^2$, $y = t - 1$

$$y = t - 1 \Rightarrow t = y + 1 \text{ substitute into } x = 3t^2$$

$$x = 3(y + 1)^2 \text{ do not expand, leave in brackets, rearrange to make } y \text{ (the squared term) the subject}$$

$$(y + 1)^2 = \frac{1}{3}x \text{ so the vertex is } (0, -1) \text{ and } 4a = \frac{1}{3} \Rightarrow a = \frac{4}{3}$$

b) $x = 2 + t^2$, $y = 3 + 2t$

$$y = 3 + 2t \Rightarrow t = \frac{1}{2}(y - 3) \text{ substitute into } x = 2 + t^2$$

$$x = 2 + \frac{1}{4}(y - 3)^2 \text{ do not expand, leave in brackets, rearrange to make } y \text{ (the squared term) the subject}$$

$$(y - 3)^2 = 4(x - 2) \text{ so the vertex is } (2, 3) \text{ and } 4a = 4 \Rightarrow a = 1$$

c) $x = 3 + 2t^2$, $y = 4t - 1$

$$y = 4t - 1 \Rightarrow t = \frac{1}{4}(y + 1) \text{ substitute into } x = 3 + 2t^2$$

$$x = 3 + \frac{1}{8}(y + 1)^2 \text{ do not expand, leave in brackets, rearrange to make } y \text{ (the squared term) the subject}$$

$$(y + 1)^2 = 8(x - 3) \text{ so the vertex is } (3, -1) \text{ and } 4a = 8 \Rightarrow a = 2$$

d) $x = t - 3$, $y = 2t^2 + 5$

$$x = t - 3 \Rightarrow t = x + 3 \text{ substitute into } y = 2t^2 + 5$$

$$y = 2(x + 3)^2 + 5 \text{ do not expand, leave in brackets, rearrange to make } x \text{ (the squared term) the subject}$$

$$(x + 3)^2 = \frac{1}{2}(y - 5) \text{ so the parabola is vertical with vertex } (-3, 5), \text{ and } 4a = \frac{1}{2} \Rightarrow a = \frac{1}{8}.$$